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A Study of Time-Domain and Frequency- Domain Techniques in Electromagnetics

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ABSTRACT
Before the advent of digital computer, analytical techniques were used for determining idealized solutions of electromagnetic problems, most especially problems of simple geometry or design. When the size of the problem becomes large that the analytical technique is unable to yield solutions of desired accuracy, approximate solutions are sought by using digital computer and numerical technique. This paper examines various numerical techniques suitable for solving electromagnetic problems. It is emphasized in the paper, the strengths and weaknesses of these techniques in tackling a particular problem. Because of the significance of retarded potentials in formulating integral equations that are solved by method of moment technique, effort is also geared towards deriving expressions for these potentials when sources are constrained to the axis, surface and volume of the conducting body.

General Terms
Computation Electromagnetics, Electromagnetic Problems, Numerical Techniques.

Keywords
Analytic technique, frequency domain technique, method of moments and time domain technique.

1. INTRODUCTION
Over the years, the accurate determination of the electromagnetic field that excites a scatterer or fields radiated by an antenna structure has been the subject of great interest to a number of investigators. In pre-computer days, exact and closed form solutions of electromagnetic problems were obtained by using analytic techniques after invoking certain assumption. Such solution exists for problems of relatively simple designs or configurations. However, when the problem becomes intractable that the analytical technique is ineffective for obtaining solution, attention is shifted to the use of high speed digital computer and numerical technique that generates approximate solution. Perhaps, the development of digital computer and introduction of several numerical techniques have extended the range of solution of radiation, scattering and waveguide modeling problems with less worry about the nature and type of the structure being analyzed. The art of utilizing digital computer and numerical technique to proffer solutions to intractable electromagnetic problems is referred to as Computational Electromagnetics (CEM).

Numerical techniques used in CEM are categorized into Numerical Methods and High frequency or asymptotic technique [1]. Numerical Methods are subdivided into Time-domain and low frequency domain techniques. They are suitable for analyzing electromagnetic structures of few wavelengths while High frequency Methods handle structures of many wavelengths.

In today's world, CEM has become an integral part of developmental process in the creation of electromagnetic devices like antenna, microwave ovens, attenuators, waveguides etc. with a lot of advantages. These advantages include the reduction of ample time designers spend in the production of these devices. The accuracy of computational techniques is such that, the production process now progresses from initial designs to final prototype without further testing [2]-[3]. CEM has also enabled the designers to view on the personal computer, the performance characteristics of these devices providing useful information than ever before. In addition, it has contributed immensely in reducing the cost of production of these devices and improving their accuracies. Application of computational electromagnetics is not only restricted to the field of electromagnetics and antenna engineering alone, it has useful applications in other areas of Electrical Engineering. It is useful in Power Engineering systems for the design and analysis of power devices like generators, transformers, insulators, turbines etc. It is also an important technique in Micro-Computing for the design of small and fast micro-processors [4].

This paper presents general description of these numerical techniques suitable for solving electromagnetic field problems. It is also considered in the paper, the strengths as well as the weaknesses of these techniques in handling electromagnetic problems. The layout of the paper is such that, section I focuses on the introduction to the subject while section II examines the time domain techniques. Section III discusses low frequency domain techniques while section IV focuses on high frequency or asymptotic techniques. Finally, section V summarizes the entire work.

2. TIME DOMAIN TECHNIQUES
The most popular time domain technique which has been used over time for analyzing scattering structure [5]-[7] and radiating structures [8]-[10] is Finite Difference Time Domain Technique. It was developed in 1966 by K.S. Yee [11]. The method provides solutions to time-dependent Maxwell's equations of the form

$$\bar{\nabla} \times \bar{H} = \bar{J} + \varepsilon \frac{\partial \bar{E}}{\partial t} \qquad (1)$$

$$\bar{\nabla} \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \qquad (2)$$

$$\bar{\nabla} . \bar{B} = 0 \qquad (3)$$

$$\bar{\nabla} . \bar{D} = \rho \qquad (4)$$

where $\left(\bar{E},\bar{H}\right)$ represent electric and magnetic fields respectively, functions of space and time, $\bar{J}$ is the current density while ρ is the charge density. $\left(\bar{B},\bar{D}\right)$ denote Magnetic and Electric flux densities respectively, and $\bar{\nabla}$ is the del operator .

Expressing eqn. (1) and (2) in rectangular coordinates, the following results

$$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}=J_x+\varepsilon\frac{\partial E_x}{\partial t} \quad (5a)$$

$$\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}=J_y+\varepsilon\frac{\partial E_y}{\partial t} \quad (5b)$$

$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=J_z+\varepsilon\frac{\partial E_z}{\partial t} \quad (5c)$$

$$\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=-\mu\frac{\partial H_x}{\partial t} \quad (6a)$$

$$\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}=\mu\frac{\partial H_y}{\partial t} \quad (6b)$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-\mu\frac{\partial H_z}{\partial t} \quad (6c)$$

in which $\left(E_x,E_y,E_z\right)$ represent $\hat{x},\hat{y},$ and $\hat{z}$ components of electric field, $\left(H_x,H_y,H_z\right)$ denote $\hat{x},\hat{y},$ and $\hat{z}$ components of Magnetic field while $\left(J_x,J_y,J_z\right)$ are the $\hat{x},\hat{y},$ and $\hat{Z}$ components of the induced current.

Finite difference time domain technique involves approximating eqns. (5a) – (5c) as well as eqns. (6a) – (6c) by finite difference equations which are often discretized at spatial and time intervals [10], where the source is initialized at given instant in time and which proceeds forward in time. The process is repeated several times until steady state is reached. It is of interest to note that, the essence of sampling at equidistant spatial and time intervals is to avoid undesirable phenomenon known as aliasing and to stabilize time-marching system [6]. The method yields desired solutions for the unknown electromagnetic fields without matrix inversion and it requires lower running time. The drawback is that, it is not computationally efficient when the number of unknowns is large, and errors arise when the matching is not properly done. To circumvent these ineptitudes, the Discrete Green Function /Finite difference Time Domain Technique (DG/FDTD), Integral Equation/Finite difference Time domain technique (IE/FDTD) and Multiresolution/FDTD are possible solutions [12].

Another time domain technique that is similar to finite difference time domain when both techniques are adopted for diffusion is transmission line modeling method [13]. They are different in that, finite difference time domain technique is a two-step technique while transmission line modeling method is a single step technique [14].

3. LOW FREQUENCY METHODS

The two known low frequency methods are the Finite Element Method and the Method of Moments. The two methods yield solutions to the electromagnetic problems by converting equations in differential or integral forms into matrix equations. Each of these methods is discussed in what follows.

3.1 Finite Element Method

It is a computational technique that solves the differential boundary value problem of the form [15]

$$L(\,g\,)=f \quad (7)$$

in which L is the differential operator, g is the quantity of interest, and f is the known excitation source. The unknown quantity is approximated by finite number of unknown coefficients which are determined by converting eqn. (7) into a set of linear equations. To formulate a set of linear equations, Ritz variational or Galerkin technique is used. While finite Element Method is suitable for solving differential equation, Method of the Moment on the hand solves integral equation. Before we proceed into the description of how the method of moment proffers solutions to a given electromagnetic problem, we discuss here the continuity equations and the auxiliary functions known as electromagnetic potentials which avail us the opportunity of formulating the integral equation suitable for method of moment implementation.

3.2 Electromagnetic Potentials and Continuity Equations of Different Material

The integral equation that arises in electromagnetic theory and whose solution is sought using method of moment technique is most often than not formulated by the use of continuity equation and retarded Potentials known as Vector Magnetic Potential and Electric Scalar Potentials. These potentials are derivable from time harmonic Maxwell's equations of the forms

$$\bar{\nabla}\times\bar{H}=\bar{J}+j\omega\varepsilon\bar{E} \quad (8)$$

$$\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H} \quad (9)$$

$$\bar{\nabla}.\,\bar{D}=\rho \quad (10)$$

$$\bar{\nabla}.\,\bar{B}=0 \quad (11)$$

in which $\left(\varepsilon,\mu,\omega\right)$ represent permittivity, permeability and angular frequency, respectively while all other symbols assume their usual physical meanings. The Maxwell's equations are complemented by constitutive relations of the forms represented by

$$\bar{D}=\varepsilon\bar{E} \quad (12a)$$

$$H=\frac{B}{\mu}=\frac{1}{\mu}\bar{\nabla}\times\bar{A} \quad (12b)$$

in which $\bar{A}$ is the Magnetic Vector Potential.

Substitution of eqn. (12b) in eqn. (9), yields the following expression in the form represented by

$$\bar{E}=-j\omega\bar{A}-\bar{\nabla}\phi \quad (13)$$

where ϕ in eqn. (13) is the Electric Scalar potential.

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The use of eqn. (13) in eqn. (8) and invoking Lorentz expression of the form $\bar{\nabla}.\bar{A} = -j\omega\mu\varepsilon\bar{\nabla}\phi$ leads to vector wave equation expressed by

$$\bar{\nabla}^2 \bar{A} + k^2 \bar{A} = -\mu \bar{J} \quad (14)$$

where $\left(\bar{\nabla}^2\right)$ represents Laplacian while k stands for propagation constant.

Analytic solution of eqn. (14) provides us with an expression for Magnetic Vector Potential in compact form which admits an expression of the form

$$\bar{A} = \frac{\mu}{4\pi} \iiint_{v'} J \; exp\left[-jkR\right]/R \; dv' \quad (15)$$

in which v' is the volume occupied by the source element dv', R is the distance from the source point to the observation point while $exp\left[-jkR\right]/R$ is the green's function.

Similarly, the expression for the Electric Scalar Potential ϕ is obtained from scalar wave equation of the form

$$\bar{\nabla}^2 \phi + k^2 \phi = -\rho/\varepsilon \quad (16)$$

which admits representation of the form

$$\phi = \frac{1}{4\pi\varepsilon} \iiint_{v'} \rho \; exp\left[-jkR\right]/R \; dv' \quad (17)$$

It is of important to stress that, eqns. (15) and (17) are expressions for the Magnetic and Electric scalar potentials of conducting body dv' whose current and charge densities $\left(J,\rho\right)$ are constrained within the volume v'. However, if current and charges are assumed to flow along the surface of the body, the Magnetic Vector Potential and Electric Scalar Potential assume forms represented by

$$\bar{A} = \frac{\mu}{4\pi} \iint_{s'} J_s \, exp\left[-jkR\right]/R \; ds' \quad (18)$$

$$\phi = \frac{1}{4\pi\varepsilon} \iint_{s'} \rho_s \; exp\left[-jkR\right]/R \; ds' \quad (19)$$

in which $\left(J_s, \rho_s\right)$ represent surface current and charge densities, respectively and s' is the surface region occupied by the conducting body ds'.

When the charge and current sources are restricted to the axes of a thin conducting body whose radius is much smaller than the wavelength and length, the Magnetic Vector Potential and Electric Scalar Potential admit representations of the forms

$$\bar{A} = \frac{\mu}{4\pi} \int_{k'} \hat{u}_{k'} \, I\left(k'\right) exp\left[-jkR\right]/R \; dk' \quad (20)$$

$$\phi = \frac{1}{4\pi\varepsilon} \int_{k'} \rho\left(k'\right) exp\left[-jkR\right]/R \; dk' \quad (21)$$

where $\hat{u}_{k'}$ is the unit vector tangential to the axis of the body, $I\left(k'\right)$ represents filamentary current, $\rho\left(k'\right)$ is the filamentary charge and dk' stands for filamentary source element.

The current and charge sources constrained within the volume, surface and axis of a conducting body are related by the continuity equations of the forms

$$\bar{\nabla}.J = -j\omega\rho \quad (22)$$

$$\bar{\nabla}.J_s = -j\omega\rho_s \quad (23)$$

$$\frac{dI\left(k'\right)}{dk} = -j\,\omega\,\rho\left(k'\right) \quad (24)$$

where equations (22) and (23) relate charge and current sources restricted to the volume and surface of the body while eqn. (24) expresses the relationship between current and charges that are limited to the axis of the body.

It is important to emphasize that, those expressions of Magnetic Vector Potentials, Electric Scalar Potentials, and continuity equations are used to formulate the fields produced by sources distributed throughout the volume, surface and line of a conducting body in form of integral equations which are solved by the Method of Moment.

3.3 Method of Moments

Harrington in 1968 published the first work on method of moment analysis of thin wire antenna [16] and his book [17] on the subject is highly referenced in the literature. The method provides an approximate solution to the boundary value problem of the form

$$E^{in} = L\left(I\right) \quad (25)$$

where E^{in} is the incident field (known excitation source), L is the integral operator and I is the unknown current distribution. The current of interest is approximated by linear combination of known basis function and unknown current coefficient whose solution is facilitated by the network circuit parameters of the form

$$\left[I\right] = \left[Z\right]^{-1}\left[V\right] \quad (26)$$

in which $\left[Z\right]$ is the impedance matrix obtained by the inner product of weighting function and integral operator on known basis function which describes the expected characteristics of unknown quantity, $\left[Z\right]^{-1}$ is the inverse of impedance matrix. $\left[V\right]$ is the known voltage matrix whose entries represent the excitation of the antenna caused by the impressed field. For the sake of computation, the excitation voltage is usually modeled by delta gap model or magnetic frill source.

Once the inverse of the impedance matrix is determined, the solution of the current distribution becomes easily available. The impedance matrix can be evaluated either by using Galerkin's technique or point matching technique. Galerkin's technique involves the use of similar basis and weighting functions while point matching technique yields solution to the current distribution by using Dirac delta function as the weighting function. Basis functions used in MoM solution are pulse function, triangular function, and piecewise sinusoidal function, otherwise called subdomain functions and entire domain functions which include Fourier series function, polynomial function. The use of pulse function leads to step approximation, while when triangular function is used, piecewise linear approximation is obtained. Galerkin's technique involves solving two integrations while Point matching technique yields solutions to the unknown current by evaluating one integral. Consequently, Point matching technique is computationally faster than Galerkin's technique.

Method of moments has been applied for treating radiation problems concerning wire antennas [18]-[20] and scatterers [21]. It involves matrix inversion and as such it requires great amount of running time. Its inability to analyze larger structure of many wavelengths is also the disadvantage of this method.

Needless to say, buoyed by the theory of method of moments technique, quite a number of commercial software tools have been developed and introduced into the market for modeling electromagnetic behaviour of wire or aperture type antennas. These software packages include Numerical Electromagnetic Code (NEC) designed by Gerald Burke and Andrew Poggi at Lawrence Livermore National Laboratory, United States of America as well Mini-Numerical Electromagnetic code (MINIEC) that was developed by Naval Ocean System Center, using Basic programming language. Several forms of NEC exist in the market which include NEC-2, NEC-3, NEC-4, and EZNEC.

Software tool that was also developed based on method of moment solution of integral equation is Feko suite. It is a simulation package designed for analyzing radiation and non-radiation problems of complex geometries. It is also useful for Electromagnetic Compatibility (EMC) analysis and for examining the radiation effects of wire cables.

4. HIGH FREQUENCY METHODS

Numerical methods discussed in the foregoing sections yields solutions of poor convergence when they are utilized for analyzing electrically large structures. High frequency methods are approximate techniques that can handle such problems and produce solutions of desired accuracy. The techniques include the Geometrical Theory of Diffraction, Uniform Theory of Diffraction, and Geometrical Optics. They have been applied to solve problems concerning electromagnetic radiation from horn, reflector antennas, antenna on aircraft as well as open ended wave guide problems. Because of the inability of Geometrical Optics to determine the diffracted fields, there came the introduction of Geometrical theory of diffraction which not only facilitates the determination of primary fields but also the diffracted fields. The method proffers solutions to electromagnetic problems as superposition of primary fields (reflected and incident fields) and diffracted fields [22]. An upgrade on the Geometrical theory of diffraction technique is the Uniform theory of diffraction which can also yield total fields of electromagnetic problems.

5. CONCLUDING REMARK

It is presented in this paper an overview of numerical techniques that can be used to obtain approximate solutions of electromagnetic problems. These techniques include time domain technique which includes the finite difference time domain technique, suitable for solving time dependent Maxwell's equations as well as low frequency techniques such as method of moments, finite element method that handle boundary value problems in integral and differential forms. The inability of aforesaid numerical techniques to yield accurate solutions for problems of complex geometry or design brings to the fore the use of high frequency methods which produce solutions of higher accuracy and convergence for radiation problems as superposition of primary and secondary fields. A number of hybrid methods are also highlighted in the paper which overcomes the weaknesses associated with the numerical techniques.

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