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A Generalized Contrast Enhancement Method for Greyscale and Colored Images

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ABSTRACT
Image contrast enhancement techniques can be used in various image processing applications. There are various Image contrast enhancement techniques available. It depends on the particular type of image, which technique will prove better. Histogram equalization is one of the best technique that can be used for contrast enhancement. Different variants of Histogram equalization like Adaptive Histogram Equalization, Contrast Limited Adaptive Histogram equalization, Multiscale Adaptive Histogram Equalization, Dynamic Histogram Equalization, Partially overlapped sub-block Histogram Equalization, etc. can also be used. All these techniques mentioned proves good for a particular type of image, but they are not generalized methods which can be used for broad variety of images. A new method of contrast enhancement called as Gray level grouping (GLG) is suggested in this paper. GLG is a generalized method and can be used satisfactorily for broad variety of images. A variant of GLG, i.e. Adaptive Gray Level grouping (AGLG) is also discussed. [4]

General Terms
Algorithm, limitations, Histogram Equalization, Contrast Enhancement

Keywords
Gray level grouping (GLG), AHE, MAHE, CLAHE, DHE, POSHE, low contrast images, bins.

1. INTRODUCTION
Image enhancement techniques, though simple, are considered the most powerful and appealing areas of image processing. Image enhancement is used to convert the original image to more suitable image that can be used for a particular application. The methods of Histogram Equalization are application specific and hence in different image processing applications a method or combinations of methods are used. Image enhancement includes gray level and contrast manipulation, noise reduction, edge-sharpening, filtering, interpolation, magnification, pseudo coloring, negative images, power transformations and so on. These are highly subjective in nature and depends on visual interpretation. Image Enhancement Techniques are mainly classified into two broad categories –

(1) Spatial domain method
(2) Frequency domain method

Spatial domain methods directly operate on the image pixels whereas frequency domain techniques are used to modify the Fourier transform of an image. Image enhancement in spatialdomain consists of point processing techniques and spatial filtering methods whereas frequency domain technique uses DFT and IDFT. [1]

2. HISTOGRAM EQUALIZATION TECHNIQUES
A good contrast image is one which has equal no. of pixels in complete range of its gray levels. Hence for getting a good contrast image our aim is to spread the dynamic range and have equal pixels in all the gray levels. This technique is known as Histogram Equalization. In following sections few traditional histogram equalization methods of contrast enhancement are discussed.

2.1 Adaptive Histogram Equalization
The global HE uses histogram information over the whole image hence it cannot adapt to local brightness features of input image. This limits contrast stretching ratio in some part of an image and causes significant losses in the background and other small areas. In AHE first the image is sub-divided into small regions and then this technique is applied to small regions (tiles) instead of the entire image. Here contrast of each tile is enhanced separately. Histogram of each output region approximately matches the specified histogram. In order to eliminate artificially induced boundaries, the neighboring tiles are then combined using bilinear interpolation. [10]

2.1.1 Algorithm
1) Divide the image into small regions (tiles)
2) Apply AHE to all tiles separately.
3) Each tile's contrast is enhanced.
4) Histogram of each output region approximately matches the specified histogram.
5) Combine the neighboring tiles by using bilinear interpolation in order to eliminate artificially induced boundaries

2.1.2 Limitations
AHE is not applicable to many images such as images with large areas of dark background. In such cases because of its adaptive nature AHE turns the black background into white and result into undesirable output. [10]

2.1 Contrast Limited Adaptive Histogram Equalization (CLAHE)
CLAHE is generalization of AHE. It has more flexibility in choosing local histogram mapping function. In CLAHE clipping level of the histogram is selected to avoid undesired noise amplification. Boundary artifacts can be reduced by method of background subtraction. [10]

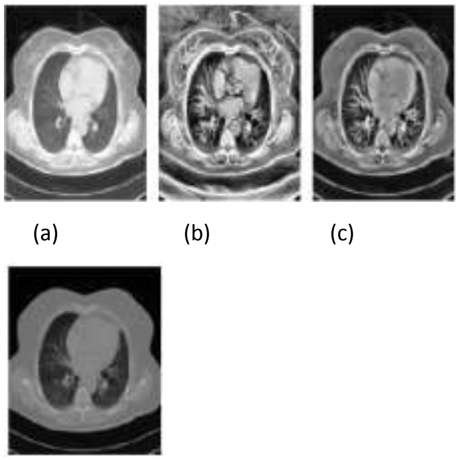
2.2.1 Algorithm
1) Divide the image into small regions.
2) Choose the local histogram mapping function.
3) Select the clipping level of histogram.

4) Apply the function to each region.

5) Noise can be reduced by the method of background subtraction.

2.2.2 *Limitations*

This method cannot be applied for broad range of low contrast image.



(a) (b) (c)

(d)

Figure1: Contrast enhancement by traditional methods. (a) Original chest CT image, and enhanced images by (b) HE, (c) AHE, (d) CLAHE, respectively. [10]

2.3 Multiscale Adaptive Histogram Equalization (MAHE)

In this technique image is decomposed into sub bands and corresponding enhancement technique is applied to high frequency sub band only. Then enhanced high frequency sub band is combined with low frequency sub band to reconstruct the output image. [10]

2.3.1 Algorithm

1) Decompose image into different frequency components.

2) Select decomposition filter.

3) Desired features of an interest can be separated from noise.

4) Selectively enhance features of interest.

5) Modify corresponding components in transform domain.

(a) (b) (c)

Figure2: Results of two enhancement methods (CLAHE & MAHE). Left column: Original image (a) Bone lesion. Center column: CLAHE, Right column: MAHE [10]

2.4 Dynamic Histogram Equalization (DHE)

DHE divides the original image histogram in to a number of sub-histograms until it ensures that no dominating portion is present in any of the newly created sub-histograms. The total available dynamic range is distributed among the sub histograms based on their dynamic range in input image and cumulative distribution (CDF) of histogram values. Then a dynamic gray level (GL) range is allocated for each sub-histogram to which its gray levels can be mapped by HE. This prevents small features of the input image from being dominated and washed out, and ensures a moderate contrast enhancement of each portion of the whole image. [8]Then separate transformation function is calculated based on the traditional HE method and then with respect to the transformation function, gray levels of input image are mapped to the output image. There are three different steps of DHE i.e. partitioning the histogram, allocating gray level ranges for each sub histogram and applying HE on each of them. [8]

2.4.1 Limitation

It results moderate contrast enhancement of an image.

2.5 Partially Overlapped Sub-Block Histogram Equalization (POSHE)

POSHE is essential to make the histogram equalization locally adaptive for higher contrast, and reduce the computation complexity. In this method, all pixels in each sub-block are histogram equalized using the sub-block's histogram. These sub-blocks are not overlapped with adjacent sub-blocks, so the computation complexity is reduced considerably. [7]

2.5.1 Algorithm

1) Define an $M×N$ -sized output image array for an input $M×N$ image and all the values are set to zero.

2) Consider a $M×N$ sub-block. To reduce computational complexity, a sub-block size is selected equal to the quotient of (the input image size)/ (a multiple of two). The sub-block origin is selected from the input image origin.

3) Perform Local histogram equalization for the sub block. Perform histogram equalization over the whole sub-block, and accumulate the results in the output image array.

4) Increase the horizontal-coordinate of the sub-block origin by the horizontal step size then repeat step 3. The vertical coordinate of the sub-block origin is increased by the vertical step size when the horizontal coordinate equals the horizontal input image size, and repeat horizontal POSHE. Repeat these steps until POSHE covers the whole input image plane.

5) After sub-block histogram equalization finished, divide each pixel value in the output image array by its sub-block histogram equalization frequency.

6) Eliminate the small blocking effect if generated at the sub-block boundaries, by using a blocking-effect reduction filter.

2.5.2 Limitation

This method is not automatic.

3. GRAY LEVEL GROUPING

The methods discussed in the previous sections yield satisfactory results, but they are not general techniques and cannot be applied to broad variety of images automatically. In order to overcome this problem another technique, called Gray Level Grouping is developed. In this method the histogram components are first group into The proper number of groups according to a certain criterion, then redistribute these groups of histogram components uniformly over the grayscale so that each group occupies a grayscale segment of the same size as the other groups. Then finally ungroup the previously grouped gray-levels. [4]

3.1 GLG Algorithm

1. Read the original image.
2. Plot histogram of original image.
3. Assign nonzero histogram components to gray level bins.
4. Record the left and right limits of the gray level interval.
5. Select the gray level bin of smallest amplitude.
6. Group it with the smaller of its two adjacent neighbors which will create a new set of gray level bins.
7. Create a look up table for gray-level transformation.
8. Apply the gray level transformation to original image.
9. Reconstruct the enhanced image. [4]

The transformation function is explained in detail as follows.

$T_{n-1}(k)=(i\text{-}\alpha\text{-}\{[R_{n-1}(i)\text{-}k]/[R_{n-1}(i) - L_{n-1}(i)] \})(N_{n-1} +1),$

$\text{for } L_{n-1}(1) = R_{n-1}(1)$

$T_{n-1}(k) =(i\text{-}\{[R_{n-1}(i)\text{-}k]/[R_{n-1}(i) - L_{n-1}(i)]\})\ (N_{n-1}+1),$

$\text{for } L_{n-1}(1) \neq R_{n-1}(1)$

3.2 GLG with Example

This algorithm is explained using the following example.

1. Read an original image. Plot histogram of an image.

2. Assign nonzero histogram components to gray level bins.

$G_n(i)=H_n(k)$ *for* $H_n(k)\neq 0,\ k=0,1,\ldots M\text{-}1,\ i=1,2,\ldots n.$

Gray levels are in the interval [0,8] and $n=5$ nonzero components. Its histogram components are

$H_5(1)=6,\ H_5(3)= H_5(4)=1,\ H_5(5)=4,\ H_5(7)=12,\ H_5(k)=0$ *for* $k=0,2,6,8.$

gray level bins after the nonzero components are assigned. $G_5(1)= 6,\ G_5(2)=1,\ G_5(3)=1,\ G_5(4)=4,\ G_5(5)=12.$

3. Record the left and right limits $L_n(i)$ and $R_n(i)$ of the gray level interval.

$L_n(i) = R_n(i) = k.$ *for* $H_n(k) \neq 0,$
for $k=0,1,\ldots M\text{-}1,\ i=1,2,\ldots n.$
$L_5(1) = R_5(1) =1$

$L_5(2) = R_5(2) =3$

$L_5(3) = R_5(3) =4$
$L_5(4) = R_5(4) =5$
$L_5(5) = R_5(5) =7.$

4. Find the first occurring smallest $G_n(i)$ and assign it to a.

$$a = \min G_n(i)$$

Let i_a is a group index corresponding to smallest of $G_n(i)$. *Here* $i_a = 2,\ a = G_5(2) = 1.$

5. Perform grouping. Group $G_n(i_a)$ is merged with smaller of its two adjacent neighbors. The set of gray level bins are created by adjusting $G_n(i)$.

$G_{n-1}(i) = G_n(i)\ ,\qquad \text{for } i=1,2\ldots,i'\text{-}1.$

$= a+b\ ,\qquad \text{for } i = i'.$

function for $k=0,1,\ldots M\text{-}1$In our example $\alpha=1$ and $N_4=2.67$ $= G_n(i+1),\quad \text{for } i= i'+1,\ i'+2,\ldots n\text{-}1$. Where

$b = \min (G_n(i_a\text{-}1),\ G_n(i_a+1));$

$i' = i_a\text{-}1,\ \text{for } G_n(i_a\text{-}1) \leq G_n(i_a+1)$
$= i_a,\ \text{otherwise.}$

6. $L_{n-1}(i) = L_n(i)\ ,\ i=1,2,\ldots i'.$
$= L_n(i+1)\ ,\ i'+1,\ i'+2,\ldots n\text{-}1.$
$R_{n-1}(i) = R_n(i)\ ,\ i=1,2,\ldots i'.$
$= R_n(i+1)\ ,\ i'+1,\ i'+2,\ldots n\text{-}1.$
New gray level bins are
$G_4(1)= G_5(1)=6$
$G_4(2)=a+b=2$
$G_4(3)=G_5(4)=4$

$G_4(4)=G_5(5)=12.$
New gray level intervals are
$L_4(1)= R_4(1)=1$
$L_4(2)= 3$
$R_4(2)= 4$
$L_4(3)= R_4(3)=5$
$L_4(4)= R_4(4)=7.$

7. This step perform mapping and ungrouping.

$N_{n-1} = M\text{-}1 / n\text{-}1$

For $L_{n-1}(1) \neq R_{n-1}(1)$

$=M\text{-}1 / n\text{-}1\text{-}\alpha$

For $L_{n-1}(1) = R_{n-1}(1)$

For $k=0,1,\ldots M\text{-}1$

A. If the gray level k falls into gray level bins $G_{n-1}(i)$ and $L_{n-1}(i) \neq R_{n-1}(i)$ Then

$T_{n-1}(k)=(i\text{-}\alpha\text{-}\{[R_{n-1}(i)\text{-}k]/[R_{n-1}(i)\text{-}L_{n-1}(i)]\})\ (N_{n-1})+1,$
$\text{for } L_{n-1}(1) = R_{n-1}(1)$

$T_{n-1}(k) = (i\text{-}\{[R_{n-1}(i)\text{-}k]/[R_{n-1}(i) - L_{n-1}(i)]\})(N_{n-1})+1,$

$\text{for } L_{n-1}(1) \neq R_{n-1}(1)$

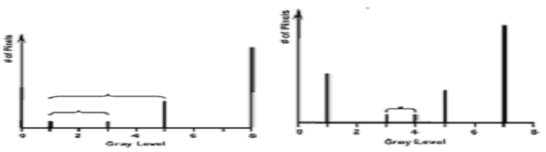
B. If gray level k falls between gray level bins $G_{n-1}(i)$ and $G_{n-1}(i+1)$ Then
$T_{n-1}(k) = (i\text{-}\alpha\ )\ N_{n-1},\quad \text{for } L_{n-1}(1) = R_{n-1}(1)$
$= i\ N_{n-1},\quad \text{for } L_{n-1}(1) \neq R_{n-1}(1)$

C. *If* $k \leq L_{n-1}(1)$ *then* $T_{n-1}(k) =0$

D. *If* $k \geq R_{n-1}(1)$ *then* $T_{n-1}(k) = M\text{-}1$

8. The transformation of gray levels are obtained.

$T_4(0)=T_4(1)=T_4(2)=0$, $T_4(3)=1$, $T_4(4)=3$, $T_4(5)=5$, $T_4(6)=5$, $T_4(7)= T_4(8)=8$.



(a) (b)

Figure3: Illustration of gray-level grouping. (a) Original histogram of a virtual low-contrast image. The bracket indicates the gray levels to be grouped. (b)Histogram after the gray-level grouping and ungrouping.[4]

4. ADAPTIVE GRAY LEVEL GROUPING (AGLG)

Similar to AHE, GLG also has its adaptive counterparts—A-GLG, or CLA-GLG. In the A-GLG or CLA-GLG method, the image is first divided into an array of sub images (usually an 8× 8 array), each sub image is processed with the GLG method, and all processed sub images are combined together by bilinear interpolation to generate the processed whole image. [4]

1. Original image is divided into an $M \times N$ array of sub images, and sub images are processed with the GLG algorithm to obtain their optimal GLG gray-level transformation functions i.e. *Ti,j(k)* for *i= 1,2,3…..M, j= 1,2,3…..N* and *k= 0,1,2….L-1* Here, *L-1* represents the highest gray-level value on the grayscale.

2. An intermediate *(M+1)×(N+1)* array of Gray-level transformation functions *Ai,j(k)* are created for *i= 1,2…M+1; j=1,2,…N+1*and *k=0,1,2…L-1* as follows:-

   a. Four corner components are calculated

$$A_{1,1}(k) = T_{1,1}(k)$$

$$A_{1,N+1}(k) = T_{1,N}(k) \qquad A_{1+1,1}(k) = T_{M,1}(k) \qquad A_{M+1,N+1}(k) = T_{M,N}(k)$$

   b. For the boundary component

$$A_{ij}(k) = \begin{cases} T_{ij}(k), & forT_{i,j-1}(k) = L-1 \\ T_{i,j-1}(k) & forT_{i,j}(k) = L-1 \\ (T_{ij-1}(k) + T_{ij}(k))/2 & otherwise \end{cases}$$

For $i = 1, M+1, j = 2,3, \dots\dots. N$, and

$k = 0,1, \dots\dots. L-1$, and

$$A_{ij}(k) = \begin{cases} T_{ij}(k), & forT_{i-1,j}(k) = L-1 \\ T_{i-1,j}(k), & forT_{i,j}(k) = L-1 \\ (T_{i-1,j}(k) + T_{ij}(k))/2, & otherwise \end{cases}$$

   c. To find the interior components

$$A_{i,j}(k) = \frac{1}{p}\sum_{m,n} T_{m,n}(k) \;\; for\; T_{m,n}(k) \neq L-1$$

Where $m = i-1, i, n = j-1, j$ and $p = 4,3,2 or 1$, It equals to the number of operands in the numerator. The above equation applies to $i = 2,3,..M, j = 2,3, \dots N, k = 0,1, \dots. L-1$ and this step is an averaging process. It balances the contrast of adjacent sub images in the final output image. If gray level k in the original image is mapped to gray-level $L-1$ by $T_{i,j}(k)$, it is considered as background and hence excluded from the averaging process.Then bilinear interpolation is performed to reconstruct the final output image. For each original sub image $I_{i,j}(x,y)$, function $k = I_{i,j}(x,y)$ returns the gray-level value of the pixel at sub image coordinate, (x,y), for $y = 1,2, \dots\dots, w_{i,j}$, $x = 1,2, \dots. h_{i,j}$, where $h_{i,j}$ and $w_{i,j}$ are the height and width of the corresponding sub image,

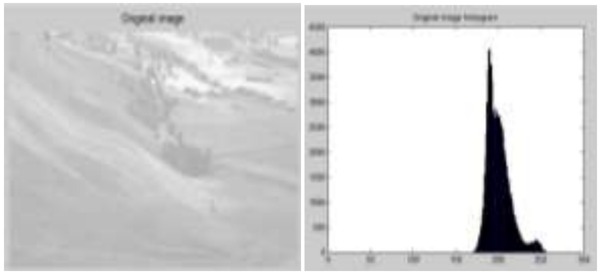
respectively. The output subimage $O_{i,j}(x,y)$ after bilinear interpolation is given by the following expression:

$$O_{i,j}(x,y) = \frac{1}{(h_{i,j}+1)(w_{i,j}+1)}$$

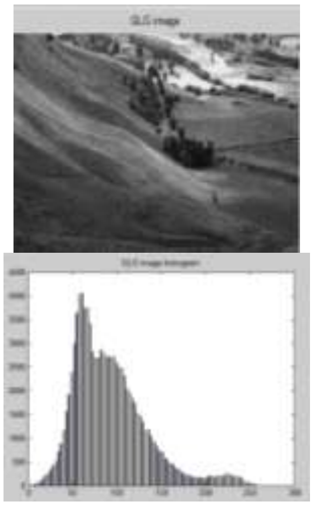
$$\times[(h_{i,j}+1-x) \qquad [(w_{i,j}+1-y)A_{i,j}(k) + yAi,j+1k$$

$$+x[(w_{i,j}+1-y)A_{i+1,j}(k) + yA_{i+1,j+1}(k)]$$

For $x = 1,2 \dots. h_{i,j}, y = 1,2 w_{i,j}$ and $k = I_{i,j}(x,y)$ the final processed whole image is obtained by combining the array of output sub images together. [4]



(a) (b)



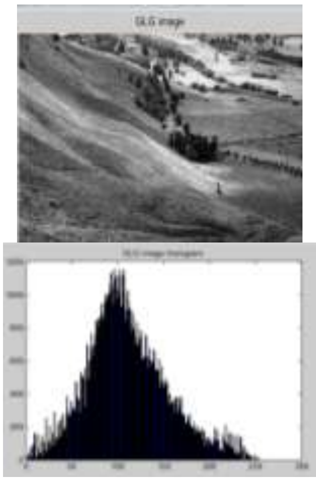


Figure 4: Results of landscape image (a) original image (D = 0.0086, TEN =0.9×10^9) (b) Global GLG result (D = 0.0489, TEN = 1.0728×10^9) (c) AGLG result (D = 0.0124, TEN = 1.6212×10^9)(d) Original image histogram (e) Global GLG image histogram (f) AGLG image histogram (Where D is average pixel distance& TEN is Tenengrad value).

The above figure (4) shows the results of landscape image. Here the image is low contrast. Its contrast is enhanced in global GLG but the result of AGLG is better as in AGLG first the image is subdivided into the sub images and then the GLG is applied, hence details of the image are more clear than the GLG image. From the figure we could analyze the changes in the histogram after processing with GLG and AGLG. The original image histogram is completely on right side of the range where as in the histogram obtained with GLG processing complete dynamic range is used.

5. Color Image Enhancement using GLG

Nowadays color images are very important as compare to greyscale images. We can use histogram equalization and various other techniques for enhancing the contrast of color image, but again they have same drawbacks as that of greyscale images. To overcome this drawback we can extend GLG technique to color images as well. To apply GLG we can use both RGB and HIS color models. For extending GLG to color images using HIS model, first decompose the image into hue, saturation and intensity components. Then apply GLG to intensity component only without changing the hue and saturation. Then after combine the hue and saturation components with processed intensity component to get the output image.For using GLG with RGB color model, first decompose the image into three components, i.e. Red, Green and Blue. Apply GLG individually to each color component. For these three components, find the maximum average distances between pixels on grayscale to determine which color component is most enhanced. Treat all color components with the transformation function of largest enhanced component. Finally combine all treated components to generate output color image.Results of GLG when applied to RGB and HSI color models are shown in figure 5 and figure 6. These results show that when GLG technique is applied to different classes of low contrast images, it gives satisfactory results. With both the approaches i.e. HSI and RGB color model, the equalized histogram is obtained and the overall contrast of the image is enhanced. Even if the contrast of image is enhanced, one should note that, there may be some color distortion occur when the image is treated with RGB color model. This happens due to the hue information is not preserved in the process. But here the overall contrast of the treated image is higher and the image is more colorful and usually more aesthetically pleasing. Hence it is recommended to use GLG to HSI model where color preservation is important and use RGB model in applications such as scientific visualization where color preservation is not very important.

6. QUALITY MEASURE

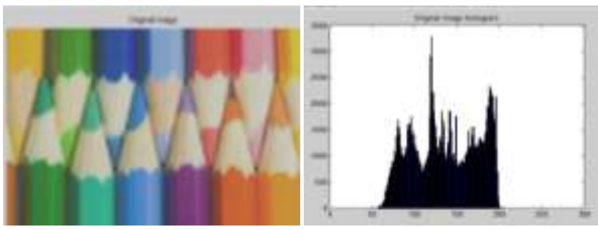
Here to analyze the quality of image, Pixel distance and Tenengrad value are the two criteria which are used. The Tenengrad criterion is based on gradient magnitude maximization. It is considered one of the most robust and functionally accurate image quality measures. The Tenengrad value of an image is calculated from the gradient $\nabla I(x,y)$at each pixel *(x,y)* were the partial derivatives are obtained by a high-pass filter, e.g. the Sobel operator, with the convolution kernels and i_x and i_y. The gradient magnitude is given as:

$$S(x,y) = \sqrt{(i_x * I(x,y))^2 + (i_y * i(x,y))^2}$$

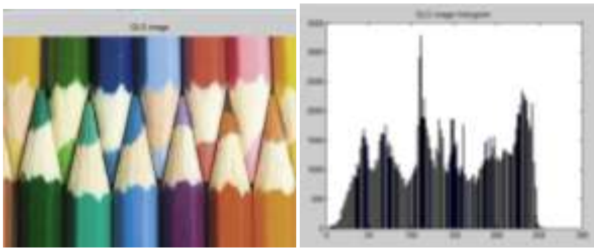
and the Tenengrad criterion is formulated as:

$$TEN = \sum_x \sum_y S(x,y)^2, \qquad for\; S(x,y) > T$$

Where T is a threshold. The image quality is usually considered higher if its Tenengrad value is larger. It is noted that the images processed with the GLG technique have significantly larger Tenengrad values, which indicates that the GLG method is superior to the conventional techniques .This result agrees with the visual evaluation by the human eye. It also should be noted that, the (PIXDIST) values of the adaptive GLG (A-GLG) results, usually do not agree with the perceived image contrasts, because the adaptive GLG process significantly changes the global histogram profile of the image, and therefore makes the comparison of the (PIXDIST) values of the global GLG and adaptive GLG results meaningless. This is one of the situations in which the PIXDIST criterion should not be used. [4]



(a) (d)



(b) (e)

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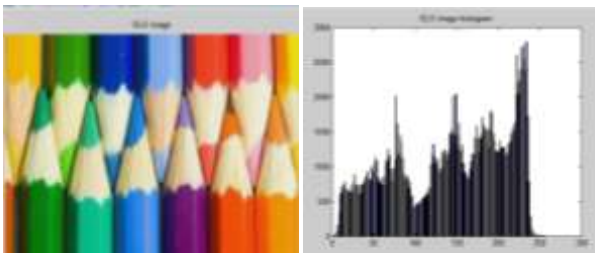
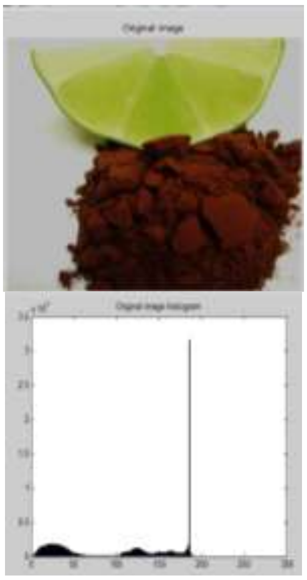


(c) (f)

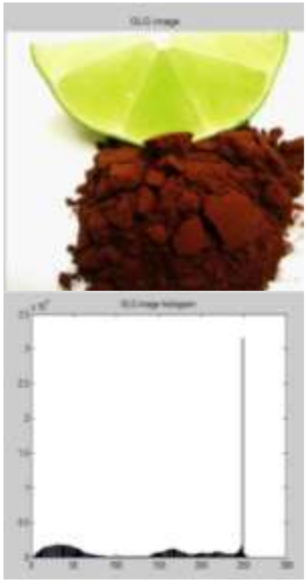
Figure 5: Results of Crayon's image (a) original image (b) GLG to HSI model (c) GLG to RGB model (d) histogram of original image (e) histogram of GLG to HIS model image (f) histogram of GLG to RGB model image.

7. CONCLUSSION & FUTURE SCOPE

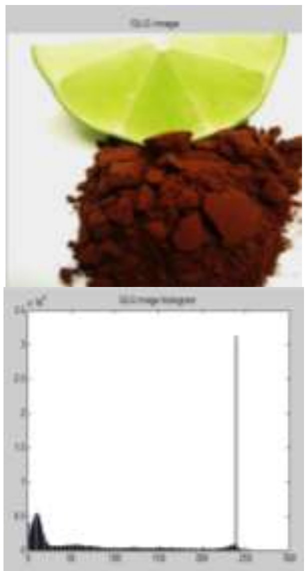
GLG is a general and powerful image enhancement technique, which can be applied satisfactorily to a broad variety of low-contrast images. The GLG technique can be applied with full automation and performs better than conventional contrast enhancement techniques. GLG has its adaptive counterpart similar to AHE. GLG can be successfully applied to color images also. In some of the cases where background noise is present, GLG does not give good results because of enhancement of background noise along with image information. To overcome this drawback, we can add preprocessing methods for background noise subtraction.



(a)



(b)



(c)

Figure 6: Result of Lemon image (a) Original image (b) GLG using HSI model (c) GLG using RGB model (d) Original image Histogram (e) Histogram of GLG using HSI model (f) Histogram of GLG using RGB model.

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