Communications on Applied Electronics (CAE) - ISSN : 2394 - 4714
Foundation of Computer Science FCS, New York, USA
Volume 3 - No. 4, November 2015 - www.caeaccess.org

The Generalized Projective Riccati Equations Method and its Applications to Nonlinear PDEs Describing Nonlinear Transmission Lines

E.M.E. Zayed
Department of Mathematics
Zagazig University
P.O.Box 44519, Zagazig, Egypt

K.A.E. Alurrfi
Department of Mathematics
Zagazig University
P.O.Box 44519, Zagazig, Egypt

ABSTRACT

In this article, we apply the generalized projective Riccati equations method with the aid of symbolic computation to construct new exact traveling wave solutions with parameters for two nonlinear PDEs describing nonlinear transmission lines (NLTL). The first equation describes the model of governing wave propagation in the NLTL as nonlinear low-pass electrical lines. The second equation describes pulse narrowing nonlinear transmission lines. The obtained solutions include, kink and anti-kink solitons, bell (bright) and anti-bell (dark) solitary wave solutions, hyperbolic solutions and trigonometric solutions. Based on Kirchhoff's current law and Kirchhoff's voltage law, the given nonlinear PDEs have been derived and can be reduced to nonlinear ordinary differential equations (ODEs) using a simple transformation. The given method in this article is straightforward and concise, and it can also be applied to other nonlinear PDEs in mathematical physics.

Keywords

Generalized projective Riccati equations method, Exact solutions, Nonlinear low-pass electrical lines, Pulse narrowing nonlinear transmission lines, Kirchhos lawsSS

1. INTRODUCTION

In the recent years, investigations of exact solutions to nonlinear PDEs play an important role in the study of nonlinear physical phenomena in such as fluid mechanics, hydrodynamics, optics, plasma physics, solid state physics, biology and so on. Several methods for finding the exact solutions to nonlinear equations in mathematical physics have been presented, such as the inverse scattering method [1], the Hirota bilinear transform method [2], the truncated Painlevé expansion method [3-6], the Bäcklund transform method [7,8], the exp-function method [9-11], the tanh-function method [12,13], the Jacobi elliptic function expansion method [14-16], the $(\frac{G'}{G})$-expansion method [17-22], the modified $(\frac{G'}{G})$-expansion method [23], the $(\frac{G'}{G}, \frac{1}{G})$-expansion method [24-27], the modified simple equation method [28-30], the multiple exp-function algorithm method [31,32], the transformed rational function method [33], the local fractional series expansion method [34], the first integral method [35,36],the generalized Riccati equation mapping method [37,38], the generalized projective Riccati equations method [39-44] and so on. Conte and Musette [39] presented an indirect method to seek more solitary wave solutions of some NPDEs that can be expressed as polynomials in two elementary functions which satisfy a projective Riccati equation [45]. Using this method, many solitary wave solutions of many NPDEs are found [42,45]. Recently, Yan [43] developed further Conte and Musette's method by introducing more generalized projective Riccati equations.
The objective of this article is to use the generalized projective Riccati equations method to construct the exact solutions of the following two nonlinear PDEs:

(1) The nonlineae PDE governing wave propagation in nonlinear low-pass electrical transmission lines [46]:

$$\frac{\partial^2 V(x,t)}{\partial t^2} - \alpha \frac{\partial^2 V^2(x,t)}{\partial t^2} + \beta \frac{\partial^2 V^3(x,t)}{\partial t^2}$$
$$-\delta^2 \frac{\partial^2 V(x,t)}{\partial x^2} - \frac{\delta^4}{12} \frac{\partial^4 V(x,t)}{\partial x^4} = 0, \qquad (1.1)$$

where α, β and δ are constants, while $V(x,t)$ is the voltage in the transmission lines. The variable x is interpreted as the propagation distance and t is the slow time. The physical details of the derivation of Eq. (1.1) using the Kirchhoff's laws are given in [46], which are omitted here for simplicity. Note that Eq. (1.1) has been discussed in [46] using an auxiliary equation method [47] and its exact solutions have been found. Also, this equation have been studied in [48] using the new Jacobi elliptic function expansion method and its exact traveling wave solutions have been obtained.

(2) The nonlinear PDE describing pulse narrowing nonlinear transmission lines [49]:

$$\frac{\partial^2 \phi(x,t)}{\partial t^2} - \frac{1}{LC_0} \frac{\partial^2 \phi(x,t)}{\partial x^2} - \frac{b_1}{2} \frac{\partial^2 \phi^2(x,t)}{\partial t^2}$$
$$-\frac{\delta^2}{12LC_0} \frac{\partial^4 \phi(x,t)}{\partial x^4} = 0, \qquad (1.2)$$

where $\phi(x,t)$ is the voltage of the pulse and C_0, L, δ and b_1 are constants. The physical details of the derivation of Eq. (1.2) is elaborated in [49] using the Kirchhoff's current law and Kirchhoff's voltage law, which are omitted here for simplicity. It is well-known [49] that Eq. (1.2) has the solution:

$$\phi(x,t) = \frac{3(v^2-v_0^2)}{b_1 v^2} \operatorname{sech}^2 \left[\frac{\sqrt{3(v^2-v_0^2)}}{v_0} \left(\frac{(x-vt)}{\delta} \right) \right], \qquad (1.3)$$

where v is the propagation velocity of the pulse and $v_0 = \frac{1}{\sqrt{LC_0}}$ provided $v > v_0$ and $LC_0 > 0$. Recently Zayed and Alurrfi have discussed Eq. (1.2) in [50] using the new Jacobi elliptic function expansion method and determined its exact traveling wave solutions.
This article is organized as follows: In Sec. 2, the description of the generalized projective Riccati equations method is given. In Sec. 3, we use the given method described in Sec. 2, to find exact solutions of Eqs. (1.1) and (1.2). In Sec. 4, physical explanations of some results are presented. In Sec. 5, some conclusions are obtained.

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2. DESCRIPTION OF THE GENERALIZED PROJECTIVE RICCATI EQUATIONS METHOD

Consider a nonlinear PDE in the form

$$P(u, u_x, u_t, u_{xx}, u_{tt}, ...) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Let us now give the main steps of the generalized projective Riccati equations method [39-44]:

Step 1. We use the following transformation:

$$u(x,t) = u(\xi), \qquad \xi = x - vt, \tag{2.2}$$

where v is velocity of the propagation , to reduce Eq. (2.1) to the following nonlinear (ODE):

$$H(u, u', u'', ...) = 0, \tag{2.3}$$

where H is a polynomial of $u(\xi)$ and its total derivatives $u'(\xi), u''(\xi), ...$ and $' = \frac{d}{d\xi}$.

Step 2. We suppose that the solution of Eq. (2.3) has the form:

$$u(\xi) = A_0 + \sum_{i=1}^{N} \sigma^{i-1}(\xi)\left[A_i\sigma(\xi) + B_i\tau(\xi)\right], \tag{2.4}$$

where A_0, A_i and B_i are constants to be determined later. The functions $\sigma(\xi)$ and $\tau(\xi)$ satisfy the ODEs:

$$\sigma'(\xi) = \varepsilon\sigma(\xi)\tau(\xi), \tag{2.5}$$

$$\tau'(\xi) = R + \varepsilon\tau^2(\xi) - \mu\sigma(\xi), \quad \varepsilon = \pm 1, \tag{2.6}$$

where

$$\tau^2(\xi) = -\varepsilon\left(R - 2\mu\sigma(\xi) + \frac{\mu^2 + r}{R}\sigma^2(\xi)\right), \tag{2.7}$$

where $r = \pm 1$ and R, μ are nonzero constants.

If $R = \mu = 0$, Eq. (2.3) has the formal solution:

$$u(\xi) = \sum_{i=0}^{N} A_i\tau^i(\xi) \tag{2.8}$$

where $\tau(\xi)$ satisfies the ODE:

$$\tau'(\xi) = \tau^2(\xi). \tag{2.9}$$

Step 3. We determine the positive integer N in (2.4) by using the homogeneous balance between the highest-order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. Substitute (2.4) along with Eqs. (2.5)-(2.7) into Eq.(2.3) or ((2.8) along with Eq. (2.9) into Eq. (2.3)). Collecting all terms of the same order of $\sigma^j(\xi)\tau^i(\xi)$ $(j = 0, 1,...; i = 0, 1)$ (or $\tau^j(\xi), j = 0, 1, \ldots$). Setting each coefficient to zero, yields a set of algebraic equations which can be solved to find the values of A_0, A_i, B_i, v, μ and R.

Step 5. It is well known [41,44] that Eqs. (2.5) and (2.6) admits the following solutions:

Case 1. When $\varepsilon = -1, r = -1, R > 0$,

$$\sigma_1(\xi) = \frac{R\text{sech}(\sqrt{R}\xi)}{\mu\text{sech}(\sqrt{R}\xi)+1}, \ \tau_1(\xi) = \frac{\sqrt{R}\tanh(\sqrt{R}\xi)}{\mu\text{sech}(\sqrt{R}\xi)+1}, \tag{2.10}$$

Case 2. When $\varepsilon = -1, r = 1, R > 0$,

$$\sigma_2(\xi) = \frac{R\text{csch}(\sqrt{R}\xi)}{\mu\text{csch}(\sqrt{R}\xi)+1}, \ \ \tau_2(\xi) = \frac{\sqrt{R}\coth(\sqrt{R}\xi)}{\mu\text{csch}(\sqrt{R}\xi)+1}. \tag{2.11}$$

Case 3. When $\varepsilon = 1, r = -1, R > 0$,

$$\sigma_3(\xi) = \frac{R\sec(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi)+1}, \ \ \tau_3(\xi) = \frac{\sqrt{R}\tan(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi)+1}, \tag{2.12}$$

$$\sigma_4(\xi) = \frac{R\csc(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi)+1}, \ \ \tau_4(\xi) = -\frac{\sqrt{R}\cot(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi)+1}. \tag{2.13}$$

Case 4. when $R = \mu = 0$,

$$\sigma_5(\xi) = \frac{C}{\xi}, \ \ \tau_5(\xi) = \frac{1}{\varepsilon\xi}, \tag{2.14}$$

where C is a nonzero constant.

Step 6. Substituting the values of A_0, A_i, B_i, v, μ and R as well as the solutions (2.10)-(2.14) into (2.4) we obtain the exact solutions of Eq. (2.1).

3. EXACT SOLUTIONS OF EQUATIONS (1.1) AND (1.2) USING THE GIVEN METHOD OF SEC. 2

In this section, we apply the generalized projective Riccati equations method of Sec. 2 to find families of new exact solutions of Eqs. (1.1) and (1.2).

3.1 Exact solutions of the nonlinear PDE (1.1)

In this subsection, we find the exact wave solutions of Eq. (1.1). To this end, we use the transformation

$$V(x,t) = V(\xi), \ \xi = \sqrt{k}\left(x - vt\right), \tag{3.1}$$

to reduce Eq. (1.1) to the following nonlinear ODE:

$$\frac{d^2}{d\xi^2}\left\{\frac{k^2\delta^4}{12}\frac{d^2V}{d\xi^2} + (k\delta^2 - kv^2)V + \alpha kv^2V^2 - \beta kv^2V^3\right\} = 0. \tag{3.2}$$

Integrating Eq. (3.2) twice and vanishing the constants of integration, we find the following ODE:

$$\frac{K^2}{12}\frac{d^2V}{d\xi^2} + (K - U)V + \alpha UV^2 - \beta UV^3 = 0. \qquad)(3.3)$$

where $K = k\delta^2$ and $U = kv^2$.
Balancing $\frac{d^2V}{d\xi^2}$ with V^3 gives $N = 1$. Therefore, (2.4) reduces to

$$V(\xi) = A_0 + A_1\sigma(\xi) + B_1\tau(\xi), \tag{3.4}$$

where A_0, A_1 and B_1 are constants to be determined such that $A_1 \neq 0$ or $B_1 \neq 0$.
Substituting (3.4) and using (2.5)-(2.7) into Eq. (3.3), the

left-hand side of Eq. (3.3) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$. Setting the coefficients of this polynomial to be zero, yields the following system of algebraic equations:

$$\sigma^3 : -U\beta A_1^3 - \frac{1}{R}\varepsilon\left(\frac{1}{6}K^2\varepsilon^2 A_1 - 3U\beta A_1 B_1^2\right)(\mu^2+r) = 0,$$

$$\sigma^2 : -\frac{\varepsilon}{R}\left(U\alpha B_1^2 - 3U\beta A_0 B_1^2\right)(\mu^2+r) - \frac{1}{12}K^2\mu\varepsilon A_1$$
$$-\frac{1}{12}K^2\mu\varepsilon A_1 - 3U\beta A_0 A_1^2 = 0,$$

$$+U\alpha A_1^2 = 0,$$

$$\sigma^2\tau : \frac{1}{R}\varepsilon\left(U\beta B_1^3 - \frac{1}{6}K^2\varepsilon^2 B_1\right)(\mu^2+r) - 3U\beta A_1^2 B_1 = 0,$$

$$\sigma : A_1(K-U) - \varepsilon R\left(\frac{1}{6}K^2\varepsilon^2 A_1 - 3U\beta A_1 B_1^2\right)$$
$$+2\varepsilon\mu\left(U\alpha B_1^2 - 3U\beta A_0 B_1^2\right) - 3U\beta A_0^2 A_1$$
$$+2U\alpha A_0 A_1 + \frac{1}{12}K^2 R\varepsilon A_1 = 0,$$

$$\sigma\tau : -2\mu\varepsilon\left(U\beta B_1^3 - \frac{1}{6}K^2\varepsilon^2 B_1\right) - \frac{1}{4}K^2\mu\varepsilon B_1$$
$$+2U\alpha A_1 B_1 - 6U\beta A_0 A_1 B_1 = 0,$$

$$\tau : B_1(K-U) + R\varepsilon\left(U\beta B_1^3 - \frac{1}{6}K^2\varepsilon^2 B_1\right)$$
$$-3U\beta A_0^2 B_1 + 2U\alpha A_0 B_1 + \frac{1}{6}K^2 R\varepsilon B_1 = 0,$$

$$\sigma^0 : A_0(K-U) - R\varepsilon\left(U\alpha B_1^2 - 3U\beta A_0 B_1^2\right)$$
$$+U\alpha A_0^2 - U\beta A_0^3 = 0. \quad (3.5)$$

Case 1. If we substitute $\varepsilon = -1$ into the algebraic equations (3.5) and solve them by Maple 14, we have the following results:

Result 1. We have

$$K = -\frac{24\alpha^2(\mu^2+r)}{(-9\beta\mu^2+2r\alpha^2+2\mu^2\alpha^2)R}, U = \frac{216\alpha^2\beta\mu^2(\mu^2+r)}{(-9\beta\mu^2+2r\alpha^2+2\mu^2\alpha^2)^2 R},$$

$$A_0 = 0, A_1 = \frac{2\alpha(\mu^2+r)}{3\beta\mu R}, B_1 = 0. \quad (3.6)$$

From (2.10), (2.11), (3.4) and (3.6), we deduce that if $r = -1$, then we have the exact wave solution

$$V(\xi) = \frac{2\alpha(\mu^2-1)}{3\beta\mu R}\left[\frac{R\operatorname{sech}(\sqrt{R}\xi)}{\mu\operatorname{sech}(\sqrt{R}\xi)+1}\right], \quad (3.7)$$

where $\xi = \sqrt{-\frac{24\alpha^2(\mu^2-1)}{\delta^2(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)R}}x - \sqrt{\frac{216\alpha^2\beta\mu^2(\mu^2-1)}{(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)^2 R}}t$, provided that $\beta(\mu^2-1) > 0$ and $9\beta\mu^2 > 2\alpha^2(\mu^2-1)$.

while if $r = 1$, then we have the exact wave solution

$$V(\xi) = \frac{2\alpha(\mu^2+1)}{3\beta\mu R}\left[\frac{R\operatorname{csch}(\sqrt{R}\xi)}{\mu\operatorname{csch}(\sqrt{R}\xi)+1}\right], \quad (3.8)$$

where $\xi = \sqrt{-\frac{24\alpha^2(\mu^2+1)}{\delta^2(-9\beta\mu^2+2\alpha^2+2\mu^2\alpha^2)R}}x - \sqrt{\frac{216\alpha^2\beta\mu^2(\mu^2+1)}{(-9\beta\mu^2+2\alpha^2+2\mu^2\alpha^2)^2 R}}t$, provided that $\beta > 0$ and $9\beta\mu^2 > 2\alpha^2(\mu^2+1)$.

Result 2. We have

$$K = -\frac{24\alpha^2}{(2\alpha^2-9\beta)R}, U = \frac{216\alpha^2\beta}{(2\alpha^2-9\beta)^2 R}, A_0 = \frac{\alpha}{3\beta},$$

$$A_1 = \pm\frac{\alpha\sqrt{\mu^2+r}}{\beta R}, B_1 = \pm\frac{\alpha}{3\beta\sqrt{R}}. \quad (3.9)$$

In this case, we deduce that if $r = -1$, then we have the exact wave solution

$$V(\xi) = \frac{\alpha}{3\beta}\left[1 \pm \frac{\sqrt{\mu^2-1}\operatorname{sech}(\sqrt{R}\xi)+\tanh(\sqrt{R}\xi)}{\mu\operatorname{sech}(\sqrt{R}\xi)+1}\right], \quad (3.10)$$

while if $r = 1$, then we have the exact wave solution

$$V(\xi) = \frac{\alpha}{3\beta}\left[1 \pm \frac{\sqrt{\mu^2+1}\operatorname{csch}(\sqrt{R}\xi)+\coth(\sqrt{R}\xi)}{\mu\operatorname{csch}(\sqrt{R}\xi)+1}\right], \quad (3.11)$$

where $\xi = \sqrt{-\frac{24\alpha^2}{\delta^2(2\alpha^2-9\beta)R}}x - \sqrt{\frac{216\alpha^2\beta}{(2\alpha^2-9\beta)^2 R}}t$, provided that $\beta > 0$ and $9\beta > 2\alpha^2$.

Case 2. If we substitute $\varepsilon = 1$ and $r = -1$ into the algebraic equations (3.5) and solve them by Maple 14, we have the following results:

Result 1. We have

$$K = \frac{24\alpha^2(\mu^2-1)}{(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)R}, U = -\frac{216\alpha^2\beta\mu^2(\mu^2-1)}{(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)^2 R},$$

$$A_0 = 0, A_1 = \frac{2\alpha(\mu^2-1)}{3\beta\mu R}, B_1 = 0. \quad (3.12)$$

From (2.12), (2.13), (3.4) and (3.12), we deduce the following exact wave solutions

$$V(\xi) = \frac{2\alpha(\mu^2-1)}{3\beta\mu R}\left[\frac{R\sec(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi)+1}\right], \quad (3.13)$$

or

$$V(\xi) = \frac{2\alpha(\mu^2-1)}{3\beta\mu R}\left[\frac{R\csc(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi)+1}\right], \quad (3.14)$$

where $\xi = \sqrt{\frac{24\alpha^2(\mu^2-1)}{\delta^2(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)R}}x - \sqrt{-\frac{216\alpha^2\beta\mu^2(\mu^2-1)}{(-9\beta\mu^2-2\alpha^2+2\mu^2\alpha^2)^2 R}}t$, provided that $\beta(\mu^2-1) < 0$ and $9\beta\mu^2 > 2\alpha^2(\mu^2-1)$.

3.2 Exact solutions of the nonlinear PDE (1.2)

In this subsection, we find the exact solutions of Eq. (1.2). To this end, we use the transformation (2.2) to reduce Eq. (1.2) to the following nonlinear ODE:

$$\phi''(\xi) + k_1\phi(\xi) + k_2\phi^2(\xi) = 0, \quad (3.15)$$

where

$$k_1 = -\frac{12(v^2-v_0^2)}{\delta^2 v_0^2}, k_2 = \frac{6b_1 v^2}{\delta^2 v_0^2}. \quad (3.16)$$

Balancing ϕ'' with ϕ^2 gives $N = 2$. Therefore, (2.4) reduces to

$$\phi(\xi) = A_0 + A_1\sigma(\xi) + A_2\sigma^2(\xi)$$

$$+B_1\tau(\xi) + B_2\sigma(\xi)\tau(\xi), \quad (3.17)$$

where A_0, A_1 , $A_2, B_1$and B_2 are constants to be determined such that $A_2 \neq 0$ or $B_2 \neq 0$.
Substituting (3.17) and using (2.5)-(2.7) into Eq. (3.15), the left-hand side of Eq. (3.15) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$. Setting the coefficients of this polynomial to be zero,

yields the following system of algebraic equations:

$$\sigma^4 : A_2^2k_2 - \tfrac{1}{R}\varepsilon\left(\mu^2+r\right)\left(6A_2\varepsilon^2+k_2B_2^2\right)=0,$$

$$\sigma^3 : 2\varepsilon\mu\left(6A_2\varepsilon^2+k_2B_2^2\right)-2\mu\varepsilon A_2+2A_1A_2k_2 \\ -\tfrac{\varepsilon}{R}\left(\mu^2+r\right)\left(2A_1\varepsilon^2+2B_1B_2k_2\right)=0,$$

$$\sigma^3\tau : 2A_2B_2k_2-\tfrac{6}{R}\varepsilon^3B_2\left(\mu^2+r\right)=0,$$

$$\sigma^2 : k_2\left(A_1^2+2A_0A_2\right)-\varepsilon R\left(6A_2\varepsilon^2+k_2B_2^2\right) \\ +2\varepsilon\mu\left(2A_1\varepsilon^2+2B_1B_2k_2\right)-\tfrac{\varepsilon}{R}B_1^2k_2\left(\mu^2+r\right) \\ +A_2k_1+2R\varepsilon A_2-\mu\varepsilon A_1=0,$$

$$\sigma^2\tau : \varepsilon\left(12\mu\varepsilon^2B_2-\tfrac{2}{R}\varepsilon^2B_1\left(\mu^2+r\right)\right) \\ +2k_2\left(A_1B_2+A_2B_1\right)-6\mu\varepsilon B_2=0,$$

$$\sigma : -\varepsilon\left(R\left(2A_1\varepsilon^2+2B_1B_2k_2\right)-2\mu B_1^2k_2\right) \\ +A_1k_1+R\varepsilon A_1+2A_0A_1k_2=0,$$

$$\sigma\tau : B_2k_1+\varepsilon\left(4\mu\varepsilon^2B_1-6R\varepsilon^2B_2\right)-3\mu\varepsilon B_1 \\ +2k_2\left(A_0B_2+A_1B_1\right)+5R\varepsilon B_2=0,$$

$$\tau : 2RB_1\varepsilon-2RB_1\varepsilon^3+B_1k_1+2A_0B_1k_2=0,$$

$$\sigma^0 : A_0^2k_2+A_0k_1-R\varepsilon B_1^2k_2=0. \quad (3.18)$$

Case 1. If we substitute $\varepsilon = -1$ into the algebraic equations (3.18) and solve them by Maple 14, we have the following results:

Result 1. We have

$$R=-k_1,\ A_0=0,\ A_1=\frac{3\mu}{k_2},\ A_2=\frac{3(\mu^2+r)}{k_1k_2},$$

$$B_1=0,\ B_2=\pm\frac{3}{k_2}\sqrt{-\frac{(\mu^2+r)}{k_1}}, \quad (3.19)$$

where $k_1 < 0, \mu^2 + r > 0$.
From (2.10), (2.11), (3.17) and (3.19), we deduce that if $r = -1$, then we have the exact wave solution

$$\phi(\xi)=-\frac{3k_1\operatorname{sech}(\sqrt{-k_1}\xi)\left(\mu+\operatorname{sech}(\sqrt{-k_1}\xi)\pm\sqrt{\mu^2-1}\tanh(\sqrt{-k_1}\xi)\right)}{k_2\left(\mu\operatorname{sech}(\sqrt{-k_1}\xi)+1\right)^2}, \quad (3.20)$$

while if $r = 1$, then we have the exact wave solution

$$\phi(\xi)=-\frac{3k_1\operatorname{csch}(\sqrt{-k_1}\xi)\left(\mu-\operatorname{csch}(\sqrt{-k_1}\xi)\pm\sqrt{\mu^2+1}\coth(\sqrt{-k_1}\xi)\right)}{k_2\left(\mu\operatorname{csch}(\sqrt{-k_1}\xi)+1\right)^2}, \quad (3.21)$$

Result 2.

$$A_0=-\frac{k_1}{k_2},\ A_1=\frac{3\mu}{k_2},\ A_2=-\frac{3(\mu^2+r)}{k_1k_2},$$

$$B_1=0,\ B_2=\pm\frac{3}{k_2}\sqrt{\frac{(\mu^2+r)}{k_1}},R=k_1. \quad (3.22)$$

where $k_1 > 0, \mu^2 + r > 0$
In this case, we deduce that if $r = -1$, then we have the exact wave solution

$$\phi(\xi)=-\frac{k_1}{k_2}\left(1+\frac{3\operatorname{sech}(\sqrt{k_1}\xi)\left(-\mu-\operatorname{sech}(\sqrt{k_1}\xi)\pm\sqrt{\mu^2-1}\tanh(\sqrt{k_1}\xi)\right)}{\left(\mu\operatorname{sech}(\sqrt{k_1}\xi)+1\right)^2}\right), \quad (3.23)$$

while if $r = 1$, then we have the exact wave solution

$$\phi(\xi)=-\frac{k_1}{k_2}\left(1+\frac{3\operatorname{csch}(\sqrt{k_1}\xi)\left(-\mu+\operatorname{csch}(\sqrt{k_1}\xi)\pm\sqrt{\mu^2+1}\coth(\sqrt{k_1}\xi)\right)}{\left(\mu\operatorname{csch}(\sqrt{k_1}\xi)+1\right)^2}\right), \quad (3.24)$$

Case 2. If we substitute $\varepsilon = 1$ and $r = -1$ into the algebraic equations (3.18) and solve them by Maple 14, we have the following results:

Result 1. We have

$$A_0=0,\ A_1=-\frac{3\mu}{k_2},\ A_2=\frac{3(\mu^2-1)}{k_1k_2},$$

$$B_1=0,\ B_2=\pm\frac{3}{k_2}\sqrt{-\frac{(\mu^2-1)}{k_1}},R=k_1, \quad (3.25)$$

where $k_1 > 0, \mu^2 - 1 < 0$.
From (2.10), (2.11), (3.17) and (3.25), we deduce the following exact wave solutions

$$\phi(\xi)=-\frac{3k_1\sec(\sqrt{k_1}\xi)\left(\mu+\sec(\sqrt{k_1}\xi)\pm\sqrt{-(\mu^2-1)}\tan(\sqrt{k_1}\xi)\right)}{k_2\left(\mu\sec(\sqrt{k_1}\xi)+1\right)^2}, \quad (3.26)$$

or

$$\phi(\xi)=-\frac{3k_1\csc(\sqrt{k_1}\xi)\left(\mu+\csc(\sqrt{k_1}\xi)\pm\sqrt{-(\mu^2-1)}\cot(\sqrt{k_1}\xi)\right)}{k_2\left(\mu\csc(\sqrt{k_1}\xi)+1\right)^2}, \quad (3.27)$$

Result 2.

$$A_0=-\frac{k_1}{k_2},\ A_1=-\frac{3\mu}{k_2},\ A_2=-\frac{3(\mu^2-1)}{k_1k_2},$$

$$B_1=0,\ B_2=\pm\frac{3}{k_2}\sqrt{\frac{(\mu^2-1)}{k_1}},R=-k_1. \quad (3.28)$$

where $k_1 < 0, \mu^2 - 1 < 0$
In this case, we deduce the following exact wave solutions

$$\phi(\xi)=\frac{k_1}{k_2}\left(-1+\frac{3\sec(\sqrt{-k_1}\xi)\left(\mu+\sec(\sqrt{-k_1}\xi)\pm\sqrt{-(\mu^2-1)}\tan(\sqrt{-k_1}\xi)\right)}{\left(\mu\sec(\sqrt{-k_1}\xi)+1\right)^2}\right), \quad (3.29)$$

or

$$\phi(\xi)=-\frac{k_1}{k_2}\left(1+\frac{3\csc(\sqrt{-k_1}\xi)\left(-\mu-\csc(\sqrt{-k_1}\xi)\pm\sqrt{-(\mu^2-1)}\cot(\sqrt{-k_1}\xi)\right)}{\left(\mu\csc(\sqrt{-k_1}\xi)+1\right)^2}\right). \quad (3.30)$$

4. PHYSICAL EXPLANATIONS OF SOME RESULTS

Solitary waves can be obtained from each traveling wave solution by setting particular values to its unknown parameters. In this section, we have presented some graphs of solitary waves constructed by taking suitable values of involved unknown parameters to visualize the underlying mechanism of the original equation. Using mathematical software Maple 14, three dimensional plots of some obtained exact traveling wave solutions have been shown in Figure 1- Figure. 6.

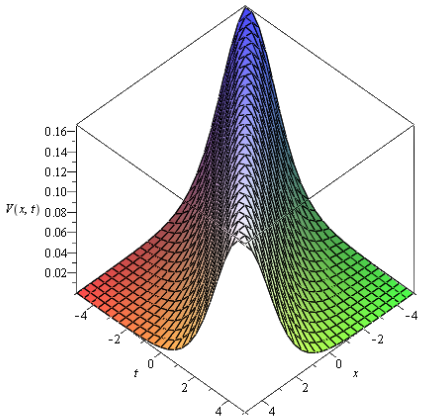

Fig. 1. The plot of the solution (3.7) when $\mu = 2$, $\alpha = 1$, $\delta = 1$, $\beta = 2$, $R = 1$.

4.1 The nonlinear PDE (1.1) governing wave propagation in nonlinear low-pass electrical transmission lines

The obtained solutions for the nonlinear PDE (1.1) incorporate three types of explicit solutions namely, hyperbolic and trigonometric. From these explicit results it is easy to say that the solution (3.7) is a bell-shaped soliton solution; the solution (3.8) is a singular bell-shaped soliton solution; the solution (3.10) is a bell-kink shaped soliton solution; the solution (3.11) is a singular bell-kink shaped soliton solution and the solutions (3.13), (3.14) are periodic solutions. The graphical representation of the solutions (3.7), (3.10) and (3.14) can be plotted as follows:

4.2 The nonlinear PDE (1.2) describing pulse narrowing nonlinear transmission lines

The obtained solutions for the nonlinear PDE (1.2) are hyperbolic and trigonometric. From the obtained solutions for this equation we observe that the solutions (3.20), (3.23) are bell-kink shaped soliton solutions; the solutions (3.21), (3.24) are singular bell-kink shaped soliton solutions and the solutions (3.26)-(3.30) are periodic solutions. The graphical representation of the solutions (3.21), (3.23) and (3.29) can be plotted as follows:

5. CONCLUSIONS

The generalized projective Riccati equations method described in Section 2 of this article has been applied to construct many new exact solutions of the nonlinear PDEs (1.1) and (1.2) which describe the nonlinear low-pass electrical transmission lines and pulse narrowing nonlinear transmission lines respectively, with the aid of Maple 14. On comparing our results obtained in this article with the well-known results obtained in [46,48,49,50] we

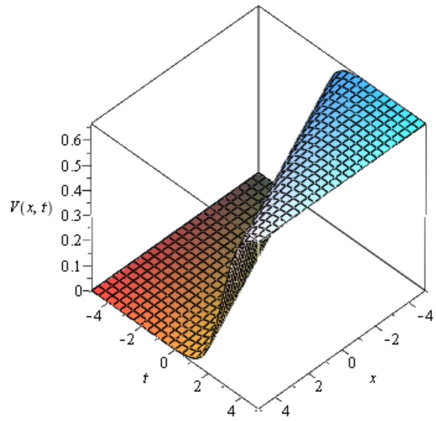

Fig. 2. The plot of the solution (3.10) when $\mu = 2$, $\alpha = 1$, $\delta = 2$, $\beta = 1$, $R = 2$.

deduce that our results are new and not published elsewhere. The proposed method of this paper is effective and can be applied to many other nonlinear PDEs. Finally, all solutions obtained in this article have been checked with the Maple 14 by putting them back into the original equations.

Acknowledgement

The authors wish to thank the referees for their comments on the paper.

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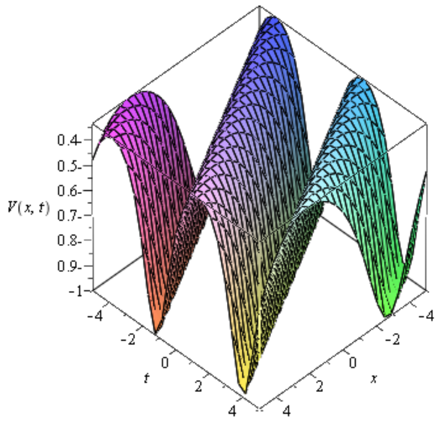

Fig. 3. The plot of the solution (3.14) when $\mu = 2$, $\alpha = 1$, $\delta = 2$, $\beta = -1$, $R = 2$.

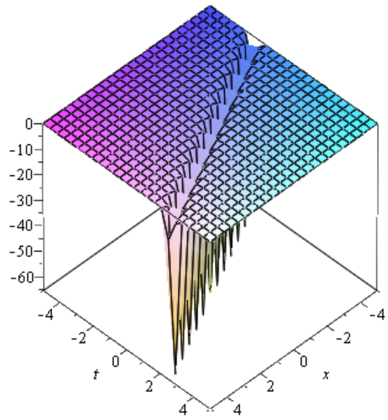

Fig. 4. The plot of solution (3.21) when $v = 2$, $\mu = 1$, $k_1 = -9$, $k_2 = 6$.

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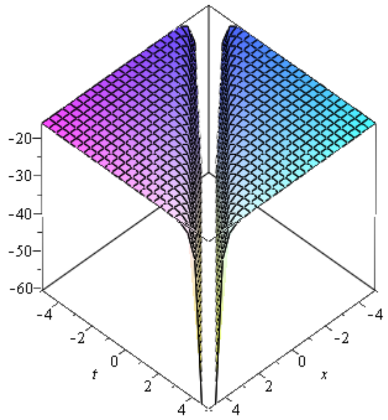

Fig. 5. The plot of solution (3.23) when $v = 1$, $\mu = -1$, $k_1 = \frac{32}{3}$, $k_2 = \frac{2}{3}$.

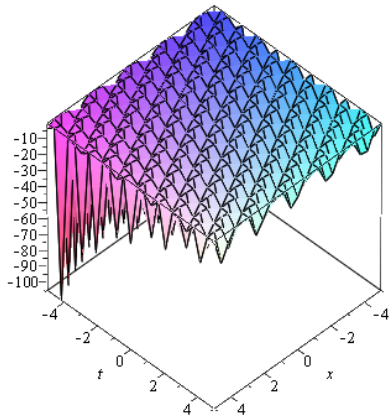

Fig. 6. The plot of solution (3.29) when $v = 2$, $\mu = \frac{1}{2}$, $k_1 = -9$, $k_2 = 12$.

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